Dr.Babasaheb Ambedkar Open University

DOR

DIPLOMA IN OPERATION
RESEARCH

Block

3

PERT & CPM

AUTHORS

1) **Shri Urvish B.Shah**
(M.Com. C.A)
I-6, Bhavna Flats,
Narayan Nagar Road, Paldi,
Ahmedabad- 07.

2) **Shri Ketul Shah**
(B.Com.C.A.)
I-1,Bhavna Flats,
Narayan Nagar Road, Paldi,
Ahmedabad- 07.

3) **Shri Mihir D. Khatri**
(B.Com.LLB. C.A)
D1/44, Arjun Tower,
Nr. C.P.Nagar, Ghatlodia,
Ahmedabad.

4) **Shri Tejas P. Shah**
(B.Com. C.A)
4, Saptsati Society,
Nr. Malav Talao, Jivraj Park,
Ahmedabad-51

5) **Shri Hareshkumar B. Vadhel**
(M.Com. M.Ed.)
B/3-401, Vishwas App. Sola Road,
Opp. Gulab Towar,
Ahmedabad.

CONTENT EDITOR

6) **Shri Krishnakant Solanki**
(B.Com., C.A)
10, Tanka Chawl,
Pajarapole, Relief Road,
Kalupur, Ahmedabad-01.

LANGUAGE EDITOR

7) **Dr. Alkesh Patel**
(M.A. B.Ed. Ph.D.)
4, Indra Apartment,
Nr. Sarvodaya School,
Sola Road, Ghatlodiya
Ahmedabad.

PUBLISHER :
Shri S. H. Barot
(I/C. Director & Registrar.)
Dr.Babasaheb Ambedkar Open University, Dafnala, Ahmedabad.
August, 2004 (Print)

Further information on the Dr.Babasaheb Ambedkar Open University courses may be obtained from the University's office at Govt. Bunglows, No-9, Dafnala, Shahibaug, Ahmedabad – 380003. Ph : (o) : 22869690, Fax : 22869691

Printed by : Shree Neminath Printers, Shahibaug, Ahmedabad. Mobile : 9825042651

Unit : 5 : PERT and CPM

Introduction

PERT means Programme Evaluation and Review Technique and CPM stands for Critical Path Method, both are related mainly with the scheduling the activities contained within the project in such a manner so that the total time used for the project is minimized. PERT (Programme Evaluation and Review Technique) was developed in 1956-58 by a research team to help in the planning and scheduling of the US Navy's Polaris Nuclear Submarine Missile project, which involved thousands of activities. Then after due to the success of the project, it was widely used in the civilian operations also. CPM (Critical Path Method) was developed by E.I. DuPont Company along with Remington Rand Corporation. In course of time, use of CPM got extended to the field of expense and resource allocation. A project is a well-defined group of actions and jobs, all of which must be completed to finish the project. Such projects involve large number of interrelated activities or tasks which must be completed in a specified time, in a specified sequence and require resources such as personnel, money, materials, facilities and/or space. So, it is mainly related with the interlinking of the activities in the project with the object of early and smooth completion of the project within the allocated time span.

Here, for solving the problem, normally graphical method is used using network for the purpose. All these techniques are based on the representation of the project as a network of activities. A network is a graphical plan consisting of a certain configuration of arrows and nodes for showing the logical sequence of various activities to be performed to achieve project objectives.

PERT or "Project Evaluation and Review Technique":

- Another derivative of the GANTT chart
- Multiple time estimates were used for each activity that allowed for variation in activity times
- Activity times are assumed to be random, with assumed probability distribution ("probabilistic")
- Activities are represented by arrowed lines between the nodes or circles

CPM or "Critical Path Method":

- Tool to analyze project and determine duration, based on identification of "critical path" through an activity network.
- Knowledge of the critical path can permit management of the project to change duration.
- A single estimate for activity time was used that did not allow for variation in activity times
- Activity times are assumed to be known or predictable ("deterministic")
- Activities are represented as nodes or circles

Structure of the Chapter:

5.1 Objectives:

By the end of the chapter the student will learn about

- Basic concepts of PERT & CPM
- Difference between PERT & CPM
- Phases of project management
- Different types of floats
- Project crashing
- Resource leveling

5.2 Basic Difference Between PERT and CPM

Following are the some of the other major differences between PERT and CPM.

PERT

It helps in identifying critical areas in a project so that necessary adjustment can be made to meet the scheduled completion date of the project. It is used for one-time projects involving activities of non-repetitive nature (i.e. activities which may never have been performed before).It assumes a probability distribution for the duration of each activity. Thus completion time estimates for all of the activities are needed. To perform PERT analysis on a project, the emphasis is given on the completion of a task.

CPM

It is used for completion of projects involving activities of repetitive nature. This technique was developed in connection with a construction and maintenance project in which duration of each activity was known with certainty. It is suitable for establishing a trade-off for optimum balancing between schedule time and expense of the project.

5.3 Significance of using PERT/CPM

It helps in timely allocation of resources to various activities to achieve optimal utilization of resources. Isolates activities, which control the project completion and therefore, results in expeditious completion of the project. It helps in the clarity of thoughts and actions and helps in clear and unambiguous communication developing from top to bottom and vice versa among the people responsible for executing the project. With the help of it delays and hold-ups during course of execution get minimized. Corrective action can also be taken well in time. Helps in the division of responsibilities and therefore, enhance effective coordination among different departments/agencies involved.

5.4 Phases Of Project Management

In general, project management consists of three phases.

Project Planning Phase

In order to visualize the sequencing or precedence requirements of the activities in a project, it is helpful to draw a network diagram. For this following tips are adopted:

1. Identify all major events and dates.
2. Decide in detail in what sequence, work packages, or tasks need to be done, and develop a network or interrelationship of tasks.
3. Estimate the duration of each separate activity. Activities should be relatable to WBS.
4. Use the activity duration estimates to calculate the estimated project duration.
5. Identify the time constraints and relate significance of each activity to timetable and major events.
6. Identify resources constraints

Scheduling Phase

The ultimate objective of the scheduling phase is to construct a time chart showing the start and finish times for each activity, as well as its relationship to other activities in the project. In addition, the schedule must pinpoint the critical (in view of time) activities that require special attention if the project is to be completed on time. For non- critical activities the schedule must show the slack or float times that can be used advantageously when such activities are delayed or when limited resources are used effectively.

The various steps involved during this phase are listed below:

1. Identify all people who will be responsible for each task.
2. Estimate the expected duration(s) of each activity, taking into consideration the resources required for their execution in most economic manner.
3. Specify the interrelationship (i.e. precedence relationship) among various activities,
4. Develop a network diagram showing the sequential interrelationship between various activities.
5. For this, tips such as; what is required to be done; why it must be done, can it be dispensed with; how to carry out the job; what must precede it; what has to follow; what can be done concurrently, may be followed,
6. Based on these time estimates, calculate the total project duration, identify critical path; calculate floats; carry out resources smoothing (or levelling) exercise for critical (or scare) resources taking into account resource constraints (if any).

Project Control Phase

PROJECT CONTROL is the final phase and network diagrams and the time chart are used for making periodic progress reports. The network must be updated and analyzed and, if necessary, a new schedule is determined for the remaining portion of the project.

Project control refers to evaluating actual progress (status) against the plan. If significant differences are observed, then the scheduling and resource allocation decisions are changed to update and revise the uncompleted part of the project. The relationship among phases of project management is shown in Given Figure, which indicates how the actual task performance data is used to track deviations from the original plan and schedule.

5.5 Pert/Cpm Network Components And Precedence Relationships

PERT/CPM network consists of two major components as discussed below:

Events

Events in the network diagram represent the start or the completion of an activity (task) or activities.

Events are commonly represented by circles (nodes) in the network diagram. The events can be further classified into following two categories:

Merge Event: An event which represents the joint completion of more than one activity is known as merge event. !

Burst Event: An event which represents the initiation (beginning) of more than one activity is known as burst event.

Note regarding Events:

1. Events in the network diagram are identified by numbers.
2. Each event should be identified by a number higher than that allotted to its

immediately preceding event to indicate progress of work.

3. The numbering of events in the network diagram must start from left (start of the project) to the right (completion of the project) and top to the bottom.
4. Care should be taken that there is no duplication in the numbering.

Activities

Activities in the network diagram represent project operations or tasks to be conducted. As such each activity except dummy consumes time and resources and incurs expenses. An arrow is commonly used to represent an activity with its head indicating the direction of progress in the project.

Activities are identified by the numbers of their starting (tail or initial) event and ending (head, or terminal) event. An arrow (i, j) between two events; the tail event i represents the start of the activity and the head evenly represents the completion of the activity.

The activities can be further classified into the following three categories:

1. Predecessor Activity: An activity which must be completed before one or more other activities start is known as predecessor activity,
2. Successor Activity: An activity which started immediately after one or more of other activities are completed is known as successor activity.
3. Dummy Activity: An activity which does not consume either any resource and/or time is known as dummy activity.

A dummy activity in the network is added only to establish the given precedence relationship among activities of the project and is needed when

Two or more parallel activities in a project have same head and tail events or

Two or more activities have some (but not all) of their immediate predecessor activities in common.

A dummy activity is depicted by dotted line in the network diagram.

Network models use following two types of precedence network to show precedence requirements of the activities in the project.

Errors and Dummies in Network

Looping and Dangling

Looping (cycling) and dangling are considered as faults in a network. Therefore, these must be avoided.

A case of disconnect activity before the completion of all activities which is also known as dangling. The dangling may be avoided by adopting rule 5 of constructing the network.

Dummy (or Redundant) Activity

The following are the two cases in which use of dummy activity may help in drawing the network correctly as per the various rules.

(i) When two or more parallel activities in a project have the same head and tail events, i.e. two events are connected with more than one arrow.

(ii) When two chains of activities have a common event, yet be wholly or partly independent of each other.

Steps to solve the sums related to PERT problems:

Step 1. Make a list of activities that make up the project including immediate predecessors.

Step 2. Making use of step 1 sketch the required network.

Step 3. Denote the most likely time by tm, the optimistic time by <o and pessimistic time by t_p.

Step 4. Using beta distribution for the activity duration, the expected time t_e, for each activity is computed by using the formula :

$t_e=(t_p + 4t_m+t_e)/6$.

Step 5. Tabulate various times, ie., expected activity times, earliest and latest times and mark the EST and LFT on the arrow diagram.

Step 6. Determine the total float for each activity by taking the difference between ESTand LFT.

Step 7. Identify the critical activities and connect them with the beginning node and the ending node, in the network diagram by double line arrows. This -gives the critical path and the expected date of completion of the project.

Step 8. Using the values of t_p and to, compute the variance (a^2) of each activity's time estimates by using the formula :

$$\sigma^2 = \left(\frac{t_p - t_o}{6}\right)^2$$

Step 9. Compute the standard normal deviate :

Z0 = Due date - Expected date of completion / Project variance

Step 10. Use standard normal tables to find the probability P ($z \leq z_0$) of completing the project within the scheduled time, where z~ N(0, 1).

5.6 Critical Path Analysis

The critical path analysis is mainly concerned with the estimation of the total project duration and to assign starting and finishing times to all activities involved in the project. This helps in checking actual progress against the scheduled duration of the project.

For the purpose, the following factors should be known to prepare project scheduling.

- Total completion time of the project.
- Earlier and latest start time of each activity.
- Float for each activity, i.e. the amount of time by which the completion of an activity can be delayed without delaying the total project completion time.
- Critical activities and critical path.

Consider the following notations for the purpose of calculating various times of events and activities.

E_i- = earliest occurrence time of an event, i. .

L_i = latest occurrence time of an event, i'.

ES_{ij} = earliest start time for an activity (i, j). .

LS_{ij} = latest start time for an activity (i,j).

EF_{ij} = earliest finish time for an activity (i, j).

LF_{ij} = latest finish time for an activity (i, J).

t_{ij}- = duration of an activity (i, j).

It has already been mentioned that in a network diagram there should only be one initial event and one end event, while other events are numbered consecutively with integer 1, 2, ...,n,- such that i <j, for any two events z and j connected by an activity which starts at i and finishes at j.

For calculating the above mentioned times, we shall discuss two methods namely forward pass method and backward pass method.

Forward Pass Method (For Earliest Event Time)

In this method, calculations begin from the initial event 1, proceed through the events in an increasing order ' of event numbers and end at the final event, say N. At each event we calculate its earliest occurrence time (E) and earliest start and finish time for each activity that begins at that event. When calculations end at .the final .event N. its earliest occurrence time gives the earliest possible completion time of the entire project. The method may be summarized as follows:

- Set the earliest occurrence time of initial event 1 to zero. That is, $E_1 = 0; i = 1$
- Calculate earliest start time for each activity that begins at event i (= 1). This is equal to earliest occurrence time of event, i (tail event).

  That is, $ES_{ij} = E_i$, for all activities (i, j) starting at event i.
- Calculate the earliest finish time of each activity that begins at event i. This is equal to the earliest start time of the activity plus the duration of the activity.

  That is, $EF_{ij} = ES_{ij} + t_{ij} = E_i + t_{ij}$, for all activities (i, j) beginning at event i
- Proceed to the next event, say j ; j > i.
- Calculate the- earliest occurrence time for the event j. This is the maximum of the earliest finish times of all activities ending into that event.

  That is, $E_J = \text{Max } \{EF_{ij}\} = \text{Max } \{E_i + t_{ij}\}$, for all immediate predecessor activities.
- If j = N (final event), then earliest finish time for the project, that is, the earliest occurrence time E_N for the final event is given by $E_N = \text{Max } \{EF_{ij}\} = \text{Max } \{E_{N-1}, + t_{ij}\}$, for all terminal activities.

Backward Pass Method (For Latest Allowable Event Time)

In this method calculations begin from final event N. Proceed through the events in the decreasing order of event numbers and end at the initial event 1. At each event, we calculate its latest occurrence time (L) and latest finish and start time for each activity that is terminating at that event and the procedure continue till the initial event. The method may be summarized as follows:

1. Set the latest occurrence of last event, N equal to its earliest occurrence time (known from forward pass method). That is, $L_N = E_N$, j = N
2. Calculate latest finish time of each activity, which ends at event j. This is equal to latest occurrence time of final event. That is, $LF_{ij} = L_j$; for all activities (i, j) ending at event j.
3. Calculate the latest start times of all activities ending at j It is obtained by subtracting the duration of the activity from the latest finish time of the activity. That is, $LF_{ij} = L_j$ and $LS_{ij} = LF_{ij} - t_{ij} = L_j - t_{ij}$, for all activity (i, j) ending at evenl j.
4. Proceed backward to the event in the sequence, which decreases j by 1.
5. Calculate the latest occurrence time of event; (i <j). This is the minimum of the latest start times of all activities from the event. That is, $L_i = \text{Min}\ \{LS_{ij}\} = \text{Min}\ \{L_j. - t_{ij}\}$, for all immediate successor activities.
6. If j=l (initial event), then the latest finish time for project, i.e. latest occurrence time L_l for the initial event is given by $L_1, = \text{Min}\{LS_{ij}\} = \text{Min}\{L_{j-1} - t_{ij}\}$, for all immediate successor activities.

5.7 Float (Slack) of an Activity and Event

The float (slack) or free time is the length of time to which a non-critical activity and/or an event can be delayed or extended without delaying the total project completion time.

Event Float

The float '(also called slack) of an event is the difference between its latest occurrence time (Z,;) and its earliest occurrence time (£,). That is,

Event float = $L_t - E_i$

Activity Float

As mentioned earlier, float is the length of free time available within the estimated times of the non-critical activities. The computation of activity float provides a measure of the extent to which non-critical activities may be extended or delayed. Mainly three types of floats are defined for each non-critical activity of the project.

Total float

It is the amount of time by which an activity can be delayed if all its preceding activities take place at their earliest possible times and following activities can be allowed to wait until there latest permissible times. The times within which an activity must be scheduled is computed from LS and £5 values for each activity's start event and end event. That is, for each activity (i,j) the total float is equal to the latest permissible end event time minus the earliest possible start event time minus the activity duration. That is,

Total float $(TF_{ij}) = (Lj - Ei) - t_{ij}$

$= LS_{ij} - Es_{ij} = Lf_{ij} - EF_{ij}$

Free float

Thus, free float for a non-critical activity is defined as the time by which the completion of an activity can be delayed without causing any delay in its immediate succeeding activities.

Free float values for each activity (I,j) are computed as follows: Free float $(FF_{ij}) = (E_j - E_i) - t_{ij}$

$= \text{Min}\ \{ES_{ij}$, for all immediate successors of activity (I,j) $\} - EF_{ij}$

Independent float

It is the amount of acceptable delay in the completion of an activity so that it neither affects its predecessor nor the successor activities. It is computed as follows:

Independent float $(IF_{ij}) = (E_j - Li.) - t_{ij} = \{ES_{ij} - LS_{ij}\} - t_{ij}$ The negative value of independent float is considered zero.

Note:

Latest occurrence time of an event is always greater than or equal to its earliest occurrence time (i.e. $Li \geq Ei$),

$TF_{ij} > (Ej - Ei) - t_{ij}$ or $TF_{ij} > FF_{ij}$

This implies that the value of free float may range from zero to total float and can never exceed total float value. That is, Independent float < Free float < Total float.

5.8 Critical Path

Certain activities in a network diagram of a project are called critical activities because delay in their execution will cause further delay in the project completion time. Thus, Critical activity: An activity whose total float is equal to zero (0)

It is the longest path starting from first to the last event and is shown by a thick line or double lines in the network diagram.

The length of the critical path is the sum of the individual times of all the critical activities lying on it and defines the minimum time required to complete the project.

The critical path on a network diagram can be identified as:

(i) For all activities (i, j) lying on the critical path the E-values and L-values for tail and head events

are equal. That, $Ej. = Lj$ and $Ei = Li$

(ii) On critical path $Ej - Ei = Lj- - Li = t_{ij}$

Finding the critical path is important for directing decision-maker's attention and effort to critical activities where improvement will pay the largest dividend.

The iterative procedure of determining the critical path is as follows :

Step 1. List all the jobs and then draw a network diagram. Each Job is indicated by an arrow with the direction of the arrow showing the sequence of jobs. The length of the arrows has no significance. Place the jobs on the diagram one by one keeping in mind what precedes and follows each job as well as what job can be done simultaneously.

Step 2. Consider the job's times to be deterministic. Indicate them above the arrow representing the task.

Step 3. Calculate the earliest start time (EST) and earliest finish time (EFT) for each event and write them in the box marked . Calculate the latest start time (LST) and latest finish time (LFT) and write them in the box marked .

Step 4. Tabulate various times, i.e.., activity normal times, earliest times and latest times, and mark EST and LFT on the arrow diagram.

Step 5. Determine the total float (slack) for each activity by taking differences between EST and LFT.

Step 6. Identify the critical activities and connect them with the beginning node and the ending node in the network diagram by double line arrows. This gives the critical path.

Step 7. Calculate the total project duration.

5.9 Project Scheduling With Uncertain Activity Times

PERT model requires three time estimates for each activity. From these times a single value is estimated for future consideration. The three-time estimates required are as under.

(i) **Optimistic time** (t_o **or a)**

This is the shortest possible time required to perform an activity, assuming that everything goes well,

(ii) **Pessimistic time (t_p or b)**

This is the longest possible time required to perform an activity under extremely bad conditions. However, such conditions do not include acts of God like earthquakes, flood, etc.

(iii) **Most likely time** (t_m **or in)**

This is the most likely time required to complete an activity. If the activity was repeated many times, then it is the duration that would occur most frequently (i.e. model value).

$$\text{Expected activity time } (t_e) = \frac{(t_o + t_p)/2 + 2\,t_m}{3} = \frac{t_o + 4\,t_m + t_p}{6}$$

Estimating the variance is apparently based on an analogy to the normal distribution where 99 per cent of the area under normal curve is within ± 3σ from the mean or fall within the range approximately 6 standard deviation in length, therefore the interval (to, tp) or range (to - tp) is assumed to enclose about 6 standard deviations of a symmetric distribution. Thus, if σ denotes the standard deviation, then

$$6\sigma \cong t_p - t_o \quad \text{or} \quad \sigma = \frac{t_p - t_o}{6}$$

Variance of activity time, $\sigma^2 = \left[\frac{1}{6}(t_p - t_o)\right]^2$

Standard deviation, $\sigma = \sqrt{\text{Variance}}$

Estimation of Project Completion Time

The probability distribution of times for completing an event can be approximated by the normal distribution due to central limit theorem. Thus, the probability of completing the project by scheduled time (T_s) is given by

$$\text{Prob}\left(Z = \frac{T_s - T_e}{\sigma_e}\right)$$

where, T_e = expected completion time of the project

Z .= number of standard deviations the scheduled time or target date lies away from the mean ' or expected date.

5.10 Project Crashing

Project crashing is a method for shortening the project duration by reducing the time of one or more of the critical project activities to less than its normal activity time.

Objective of Crashing

To reduce the scheduled completion time to reap the results of the project sooner.
As project continues over time, the team consumes indirect costs.
There may be direct financial penalties for not completing a project on time.

Key Terms

CRASHING is reducing project time by expending additional resources.
CRASH TIME is an amount of time an activity is reduced.
CRASH COST is the cost of reducing activity.

Project Crashing

The goal of crashing is to reduce project duration at minimum cost.
To reduce project duration while minimizing the cost of crashing, the project team should estimate require time, require the cost, crash time, crash cost for each activities. And then the team can estimate total crash time, total crash cost, the crash cost per week to reduce project duration at minimum cost.

Time-Expense Trade-Off Procedure

The method of establishing time-expense trade-off for the completion of a project can be summarized as follows:

Step 1

Determine the normal project completion time and associated critical path for the following two cases:

(i) When all critical activities are completed with their normal time. This provides the starting point for crashing analysis.

(ii) When all critical activities are crashed. This provides the stopping point for crashing analysis.

Step 2

Identify critical activities and compute the expense slope for each of these by using the relationship

$$\text{Expense slope} = \frac{\text{Crash expense - Normal expense}}{\text{Crash time - Normal time}}$$

The values of expense slope for critical activities indicate the direct extra expense required to execute an activity per unit of time.

Step 3

For reducing total project completion time, identify and crash an activity time on the critical path with lowest expense slope value to the point where

(i) Another path in the network becomes critical, or

(ii) The activity has been crash to its lowest possible time.

Step 4

If the critical path under crashing is still critical, return to Step 3. However, if due to crashing of an activity time in Step 3, other path(s) in the network also become critical, then identify and crash the activity(s) on the critical path(s) with the minimum joint expense slope.

Step 5

Terminate the procedure when each critical activity has been crashed to its lower possible time. Determine total project expense (indirect expense plus direct expense) corresponding to different project durations.

5.11 Resource Allocation

When resources such as men, money, material, machinery, etc., are limited and conflicting demands are made for the same type of resources as project progresses, a systematic method for the allocation of resources becomes essential. The aim is to prevent the day-to-day fluctuation in the level of required resources and obtain a uniform resource requirement during the project duration.

5.12 Resource Leveling

Resource leveling should be carried out to ensure that the project never demands more resources than have been specified as being available. This typically involves the delaying of certain tasks - in expectation of the release of resources being used. Resource leveling aims to keep all resource demands within the resource availability profiles. However if this has implications - such as the extension of the end-date of the project then these profiles may require adjustment.

The following two general rules are normally used in scheduling non-critical activities.

1. The total float of a non-critical activity is equal to its free float, then it can be scheduled anywhere between its earliest start and latest completion times
2. If the total float of a non-critical activity is more than its free float, then its starting time can be delayed relative to its earliest start time by no more than the amount of its float without affecting the scheduling of its immediately succeeding activities.

5.13 Resource Smoothing

Resource smoothing is intended to smooth out the demand for each resource. Resource smoothing can be facilitated by a variety of different techniques. Adjusting the timing of activities within their float, by the moving of allocated resources between activities. The procedure of carrying out resource smoothing can be summarized in the following steps.

Step 1

Calculate the earliest start and latest finish times of each activity and then draw a time scaled version (or squared) of the network. In this network critical path is drawn along a straight line and non-critical activities on both sides of this line. Resource requirement of each activity is given along the arrows.

Step 2

Draw the resource histogram by taking earliest start times or latest start times of activities on the x-axis and cumulative resource required on y-axis.

Step 3

Shift start time of non-critical activities first having largest float in order to smoothen the resources?

Exercise

1. Describe the concepts of PERT & CPM? State main points of difference in between them?
2. Describe the various stages of project management?
3. Describe various types of floats?
4. Write a short note on Resource smoothing and Resource leveling?

Practical Exercise:

1. Listed in the table are the activities and sequencing necessary for a maintenance job on the heat exchangers in Rajat Ltd.

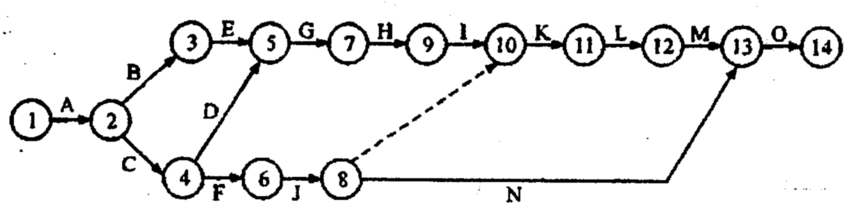
Activity	Description	Predecessor Activity
A	Dismantle pipe connections	—
B	Dismantle heater, closure, and floating front	A
C	Remove tube bundle	B
D	Clean bolts	B
E	Clean heater and floating head front	B
F	Clean tube bundle	C
G	Clean shell	C
H	Replace tube bundle	F, G
I	Prepare shell pressure test	D, E, H
J	Prepare tube pressure test and reassemble	I

Draw a network diagram for the project.

Solution The network diagram for the project is shown in Given Figure.

Draw a network diagram for the project.

Solution: The network diagram for the project is shown in Given Figure.



Given Figure Network Diagram

2. Stone, an insurance company has decided to modernize and refit one of its branch offices. Some of the existing office equipments will be disposed of but the remaining will be returned to the tranch of completion of the renovation work. Tenders are invited from a number of selected contractors. The contractors will be responsible for all the activities in connection with the renovation work excepting the prior removal of the old equipment and its subsequent replacement.

The major elements of the project have been identified as follows along with their durations and immediately preceding elements.

Activit y	Description	Duration (weeks)	Immediate Predecessors
A	Design new premises	14	—
B	Obtain tenders from the contractors	4	A
C	Select the contractor	2	B
D	Arrange details with selected contractor	1	C
E	Decide which equipment is to be used	2	A
F	Arrange storage of equipment	3	E
G	Arrange disposal of other equipment	2	E
H	Order new equipment	4	E
I	Take delivery of new equipment	3	H, L
J	Renovations take place	12	K
K	Remove old equipment for storage or disposal	4	D, F, G
L	Cleaning after the contractor has finished	2	J
M	Return old equipment for storage	2	H, L

(a) Draw the network diagram showing the inter-relations between the various activities of the project.

(b) Calculate the minimum time that the renovation can take from the design stage.

(c) Find the effect on the overall duration of the project if the estimates or tenders can be obtained in two weeks from the contractors by reducing their numbers.

(d) Calculate the 'independent float' that is associated with the non-critical activities in the network diagram.

Solution (a) The network diagram for the given project along with E - values and L - values is shown in Given Figure.

The critical path in the network diagram (Fig 13.13) has been shown by double lines joining all those events where E-values' and L-values are equal.

(b) The critical path of the project is: 1-2-3-4-7-8-9-10-12 and critical activities are A, B, C, D, K,-J, L and I. The total project completion time is 42 weeks.

For non-critical activities, the total float, free float and independent float calculations are shown in following Table .

Calculation of Floats

		Earliest Time		Latest Time			Float	
Activity	Duration	Start (Ei)	Finish (Ei + tij)	Start (Lj - tij	Finish Lj	Total (Lj - tij) - Ei	Free (Ej-Ei)-tij	Independent (Ej-Li)-tij
2- 5	2	14	16	16	18	2	0	0
6- 1	3	16	19	18	21	2	2	0
5- 7	2	16	18	19	21	3	3	1
5 - 10	4 .	16	20	35	39	19	19	17
10-11	2	39	41	40	42	1	1	1

(c) The effect on the overall project duration if the time of activity B is reduced to 2 weeks instead of4 weeks is shown in following Table.

	Path Duration	
(i)	A - E - H - I	23
(ii)	A - E - H - M	22
(iii)	A-B-C-D-K-J-L-I	
	(Critical path 42 weeks)	40 (New critical path)
(iv)	A - B - C - D - K - J - L - M , V	
	(Critical path 41 weeks)	39
(v)	A - E - G - K - J - L - I	39
(vi)	A-E-G-K-J-L-M	39
(vii)	A-.E-F-K-J-L-I	40 (New critical path)
(viii)	A-E-F-K-J-L-M	39

3. A project is-represented by the network shown below and has the following data for Marce Ltd.:

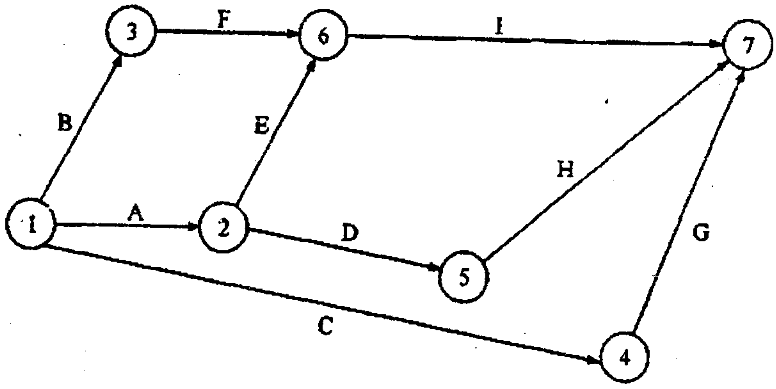
Task	A	B	C	D	E	F	G	H	I
Optimistic time	5	18	26	16	15	6	7	7	3
Pessimistic time	10	22	40	20	25	12	12	9	5
Most likely time	8	20	33	18	20	9	10	8	4

Determine the following:

(a) Expected task times and their variance.

(b) The earliest and latest expected completion times of each event.

(c) The critical path.



(d) The probability of an event occurring at the expected completion date if the original scheduled time of completing the project is 41.5 weeks.

(e) The duration of the project that will have 95 per cent chance of being completed.

Solution (a) Using the following formula, the expected activity times (te or μ) and variance ( ) is given in the following table

$$t_e = \frac{1}{6}(t_o + 4t_m + t_p) \quad \text{and} \quad \sigma^2 = \left\{\frac{1}{6}(t_p - t_o)\right\}^2$$

(b) The earliest and latest expected time for each event will be calculated by considering the expected time of each activity as shown in following Table.

Activity	t0	tp	tm	t_e= (tp + 4tm + t6)/6	
1 -2	5	10	8	7.8	0.696
1 - 3	18	22	20	20.0	0.444
1-4	26	40	33	33.0	5.429
2-5	16	20	18	18.0	0.443
2 .- 6	15	25	20	20.0	2.780
3-6	6	12	9	9.0	1.000
4-7	7	12	10	9.8	0.694
5-7	7	9	8	8.0	0.111
6-7	3	5	4	4.0	0.111

Forward Pass Method

$E_1 = 0$

$E_2 = E_1 + t_{1,2} = 0 + 7.8 = 7.8$

$E_3 = E_1 + t_{1,3} = 0 + 20 = 20$

$E_4 = E_1 + t_{1,4} = 0 + 33 = 33$

$E_5 = E_2 + t_{2,5} = 7.8 + 18 = 25.8$

$E_6 = \text{Max }\{E_i + t_{i,6}\}$

$= \text{Max }\{E_2 + t_{2,6}\,;\ E_3 + t_{3,6}\}$

$= \text{Max }\{7.8 + 20;\ 20 + 9\} = 29$

$E_7 = \text{Max }\{E_i + t_{i,7}\}$

$= \text{Max }\{E_5 + t_{5,7}\,;\ E_6 + t_{6,7}\,;\ E_4 + t_{4,7}\}$

$= \text{Max }\{25.8 + 8;\ 29 + 4\ ;\ 33 + 9.8\} = 42.8$

Backward Pass Method

$L_7 = E_7 = 42.8;$

$L_6 = L_7 - t_{6,7} = 42.8 - 4 = 38.8$

$L_5 = L_7 - t_{5,7} = 42.8 - 8 = 34.8$

$L_4 = L_7 - t_{4,7} = 42.8 - 9.8 = 33$

$L_3 = L_6 - t_{3,6} = 38.8 - 9 = 29.8$

$L_2 = \text{Min }\{L_j - t_{2,j}\}$

$= \text{Min }\{L_6 - t_{2,6}\,;\ L_5 - t_{2,5}\}$

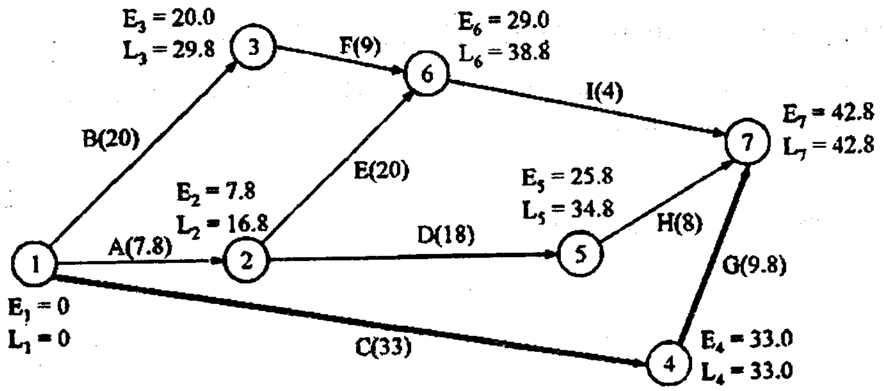
$= \text{Min }\{38.8 - 20;\ 34.8 - 18\} = 16.8$

$L_1 = \text{Min }\{L_j - t_{1,j}\}$

$= \text{Min }\{L_3 - t_{1,3}\,;\ L_4 - t_{1,4}\,;\ L_2 - t_{1,2}\}$

$= \text{Min }\{29.8 - 20\,;\ 33 - 33\,;\ 16.8 - 7.8\} = 0$

The E-value and L-values are shown in Given Figure.



Network Diagram

(c) The critical path is shown by thick line in Given Figure where E-values and L-values are same. The critical path is: 1-4-7 and the earliest completion time for the project is 42.8 weeks.

(b) The last event 7 will occur only after 42.8 weeks. For this, we require only the duration of critical activities. This will help us in calculating the standard of the duration of the last event.

Expected length of critical path = 33 + 9.8 = 42.8.

Variance of critical path length = 5.429 + 0.694 = 6.123

It is given that Ts = 41.5, Te = 42.8 and $\sigma_{e} = \sqrt{16.123} = 2.474$. Therefore, probability of g the schedule time is given by

Prob ($Z \leq$ Ts – Te / σ_e) = Prob ($Z \leq -0.52$)

= 0.30 (from normal distribution table)

Thus, the probability that the project can be completed in less than or equal to 41.5 weeks is 0.30. In other words, probability that the project will get delayed beyond 41.5 weeks is 0.70.

(e) Given that $P(Z \leq Ts - Te / \sigma_e) = 0.95$

But $Z_{0.95} = 1.64$, from normal distrbution table. Thus

$1.64 = Ts - 42.8 / 2.47$ or $T = 1.64 \times 2.474 + 42.8 = 46.85$ weeks.

4. The following table gives data on normal time, and expense and crash time and expense for a project of Kupi Ltd.

Activity	Normal		Crash	
	Time(weeks)	Expense (Rs)	Time (weeks')	Expense (Rs)
1-2	3	300	2	400
2-3	3	30	3	30
2-4	7	420	5	580
2-5	9	720	7	810
3-5	5	250	4	300
4-5	0	0	0	0
5-6	6	320	4	410
6-7	4	400	3	470
6-8	13	780	10	900
.7-8	10	1000	9	1200

Indirect expense is Rs 50 per week

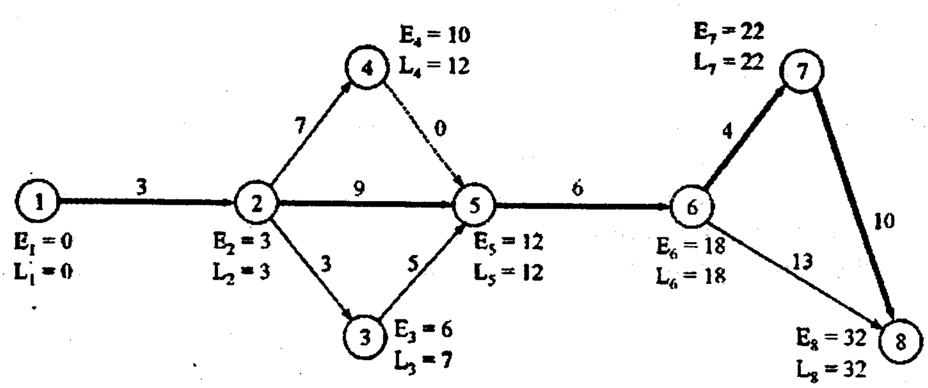
(a) Draw the network and indentify the critical path with a double line.

(b) What are the normal project duration and associated expense?

(c) Find out the total float associated with each activity.

(d) Crash the relevant activities systematically and determine the optimal project completion time and Expense.

Solution (a) The network for normal activity times indicates a project completion time of 32 weeks with the critical path: 1-2-5.-6-7-8, as shown in Given Figure.



Given Figure Network Diagram

(b) Normal project duration is 32 weeks and the associated expense is as follows: Total expense = Direct normal expense + Indirect expense for 32 weeks

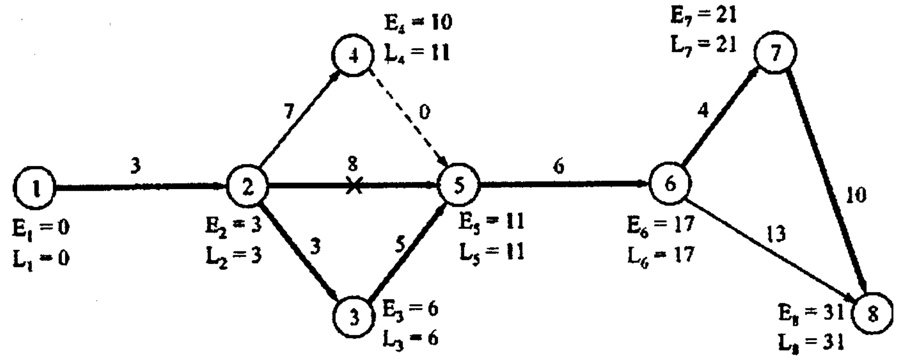
= 4,220 + 50 x 32 = Rs 5,820

(c) Calculations for total float associated with each activity are shown in following Table

.

Activity	Total Float $(L_i - E_j) - t_{ij}$
1 -2	(3 - 0) - 3=0
2-3	(7 - 3) - 7=1
2-4	(12- 3)- 7 = 2
2-5	(12 - 3) - 9 = 0
3 - 5	(12 - 6) – 5=1
4-5	(12 - 10) - 0 = 2
5-6	(18-12)- 6 = 0
6-7	(22 - 18) - 4 = 0
6-8	(32 - 18) - 13 = 1
7-8	(32 - 22) - 10 = 0

(d) For critical activities, crash expense-slope is given in Table following table.



Given Figure Network Diagram

The minimum value of crash expense per week is for activity 2-5 and 5-6. Hence, crashing activity 2 - 5 by 2 days from 9 weeks to 7 weeks. But the time should be reduced by 1 week only otherwise another path 1-2-3-5-6-7-8 become a paralled path. Network as shown in Given Figure is developed when it is observed that new project time is 31 weeks and the critical paths are 1 -2-5-6-7-8 and 1_2-3-5-6-7-8.

Critical Activity	Crash Expense per Week (Rs)
1 -2	(400-300)/3-2 =100
2- 5	(810-720)/(9-7)=45
5-6	(410-320)/(6-4) =45
6-7	(470-400)/(4-3) = 70
7-8	(1200- 1000) /(10-9)=200

With crashing of activity 2-5, the new total expense involved can be calculated as follows:

New total expense = Total direct normal expense + Increased direct expense due to crashing of activity (2 - 5) + Indirect expense for 31 weeks

$= (4,220 + 1 \times 45) + 31 \times 50 = 4,265 + 1,550 =$ Rs 5,815

Now with respect to network given in Given Figure, the new possibilities for crashing in the critical paths are listed in following table.

Critical Activity	Crashed Expense per Week (Rs)
1 -2	100
2 - 5	× (Crashed)
2-3	0 (No crashing is needed)
3-5	50
5-6	45
6-7	70
7-8	200

The minimum value of crashed expense slope is for activity 5 - 6. Hence, crashing it by 2 weeks from 6 weeks to 4 weeks. The new network diagram will now look like as shown in Given Figure.

It may be noted in Given Figure, that both the critical paths shown in Given Figure remain unchanged because activity 5 - 6 is common between critical paths shown in Given Figure. But with this crashing of activity 5 — 6 by 2 weeks, the new expense involved is:

New total expense = Total direct normal expense + Increased direct expense due to crashing of 5 - 6 + Indirect expense for 29 weeks .

$$= (4{,}220+1 \times 45+2 \times 45)+ 29 \times 50 = \text{Rs}5805$$

With respect to network given in Given Figure, the new possibilities for crashing in the critical paths are listed in Given table.

Critical Activity	Crashed Expense per Week (Rs)
1 - 2	100
2-3	0 (No crashing is needed)
2-5	× (crashed)
5-6	× (crashed)
6-7	70
7-8	200

The further crashing of 6 - 7 activity time from 4 weeks to 3 weeks will result in increased direct expense than the gain due to reduction in project time. Hence, here we must stop further crashing. The optimal project duration is 29 weeks with associated expense of Rs 5,805 as shown in following table.

Crashing Schedule of the Project

Project Duration (weeks}	Crashing Activity and Weeks	Direct Expense (Rs) Normal Crashing Total	Indirect Expense (Rs)	Total Expense (Rs)
32	—	4,220 — 4,220	32 × 50 = 1,600	5,820
31	2-5(1)	4,220 1 × 45 = 45 4,265	31 × 50 = 1,550	5,815
29	5 - 6(2)	4,220 45 + 2 × 45 = 135 4,355	29 × 50 = 1,450	5,805
28 .	6-7(1)	4,220 135 + 1 × 70 = 205 4,425	28 × 50 = 1,400	5,825 -

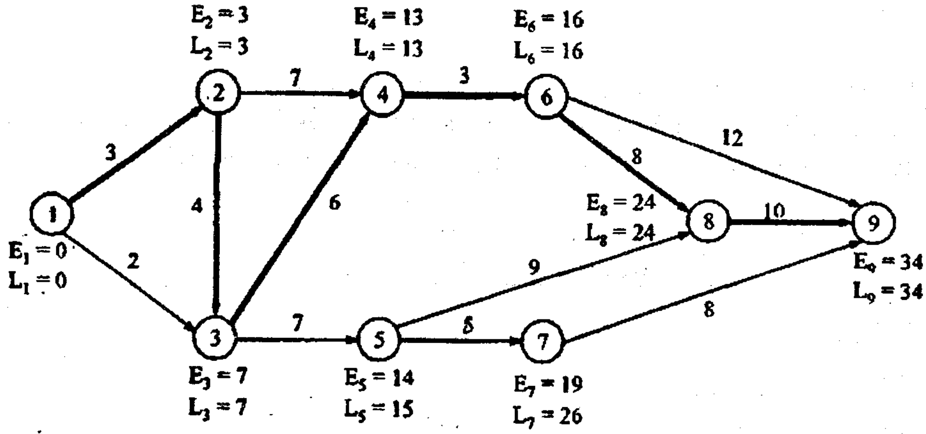
5. The network for a project in Samir Ltd. is as shown below. A review of the project after 15 days reveals that:

(a) Activities 1-2, 1-3,2-3,2-4 and 3-4 are completed.

(b) Activities 3-5 and 4 - 6 are in progress and need 2 and 4 days more respectively.

(c) The revised estimate shows that activity 8-9 will take only 8 days but 7 - 9 will need 10 days.

Draw a new network after updating the project and determine the new critical path. The work completed may be shown by an elapsed time activity.



Solution The progress of the work noted at the end of 15th day from the start of the project may be summarized as shown in following Table.

Activity	Time Required (days) Job Status	
1 - 2	0	Complete
1 -3	0	Complete
2-3	0	Complete
2-4	0	Complete
3-4	0	Complete
3 - 5 (20 - 25)	2	In progress
4 - 6 (20 - 26)	4	In progress
5 - 8 (25 - 28)	9	Not started
5 - 7 (25 - 27)	5	Not started
6 - 8 (26 - 28)	8	Not started
6 - 9 (26 - 29)	12	Not started
7 - 9 (27 - 29)	10	Not started
8 - 9 (28 - 29)	8	Not started

In the network diagram as shown in Given Figure, activity 1 - 20 shows the elapsed time of 15 days. Other activities are assigned the time which are required for their completion after review as given in Table 13.15. Based on these revised time estimates, the critical path: 1 - 20 - 26 - 28 - 29 is shown by thick lines in Given Figure. The total project duration has increased by one day.

Exercise:

1. In Naresh Ltd., an assembly is to be made from two parts X and Y. Both parts must be turned on a lathe and Y must be polished whereas X need not be polished. The sequence of activities together with their predecessors is given below.

Activity	Description	Predecessor Activity
A	Open work order	
B	Get material for X	A
C	Get material for Y	A
D	Turn X on lathe	B
E	Turn Y on lathe	B, C
F	Polish Y	E
G	Assemble X and Y	D, F
H	Pack	G

Draw a network diagram for the project.

2. Draw network diagrams from, the following list of activities for Sanket Ltd.:

Activity	Predecessor Activity		
	Set 1	Set 2	Set3
A	—	—	—
B	—	—	—
C	—	—	—
D	A	A	A
E	B	A, B	A, B
F	B, C	A, B, C	B, C
G	D, E, F	D, E, F	C
H	E, F	F	D, E,F

3. Following is the list of activities associated with the assembly of a space module for an upcoming mission. Draw the network diagram for the project for Bhakti Ltd.

Activity	Description Predecessor Activity	
A	Construct shell of module	—
B	Order life support system and scientific	
	experimentation package from supplier	—
C	Order components of control and navigation system	—
D	Wire module	A
E	Assemble control and navigational system	C
F	Preliminary test of life support system	B
G	Install life support system in module	D, F
H	Install scientific experimentation package in module	D, F
I	Preliminary test of control and navigational system	E, F
J	Install control and navigational system in module	H, I
K	Final testing and debugging	'G, J

4. A new type of water pump is to be designed for an automobile in Shilpa Ltd. Major specifications are given in the table. Draw the network diagram for the project.

Activity	Description Predecessor Activity	
A	Drawing prepared and approved	—
B	Expense analysis	A
C	Tool feasibility (economics)	A
D	Tool manufactured	C
E	Favourable expense	B, C
F	Raw materials procured	D, E
G	Sub-assemblies ordered	E
H	Sub-assemblies received	G
I	Parts manufactured	D, F
J	Final assembly	1, H
K	Testing and shipment	K

5. Listed in the table are the activities and sequencing required in the computerization of Uco bank branch.

Activity	Description ,	Predecessor Activity
A	Preparation/Deliberations in the department	—
B	Dialogue with the union	A
C	Discussion/Approval/Sanction of local management	A
D	Customer education	B
E.	Preparing specifications for the system	C
F	Selection of staff	B
G	Order/Acquisition of system	D. E
H	Alterations in the branch premises	C;
I	Training of staff	F
J	Wiring/Power-supply set-up of the branch	H
K	Transition	G, I, J
L.	Parallel run	K

Draw the arrow diagram of activities of the above project.

6. The medical faculty of a Ambedkar Open Uuniversity is considering to hold a faculty development programme. It has planned the .following activities. Prepare a network diagram showing the inter-relationships of the various activities.

Activity	Description	Predecessors
A	Design conference meetings and theme	—
B	Design front cover of the conference proceedings	A
C	Prepare brochure and send request for papers	A
D	Compile list of distinguished speakers/guests	A
E	Finalize brochure and print it	C, D
F	Make travel arrangements for distinguished speakers/guests	D
G	Despatch brochures	E
H	Receive papers for conference	G
I	Edit papers and assemble proceedings	F,H
J	Print proceedings	B, I

7. Activities and their description associated with the Rakta Watershed Project are listed below:

Activity	Description	Predecessor Activity
A	Consult General Water Board (GWB)	—
B	Motivate farmers	A
C	Identify beneficiaries spots	B
D	Obtain demand certificate from GWB	C
E	No objection certificate from GWB	C
F	Apply to DPAP	C
G	Forward to bank	D, E, F
H	Activity application	G
I	. Requisition drilling high	G
J	Release loans	H
K	Apply for power	H'
L	Purchase pump sets	I, J
M	Drilling of walls	I, J
N	Construct storage tank	I, J
O	Prepare field channels	I, J
P	Test water for quality	M
Q	Install pump sets	L, P
R	Power connections	K
s	Tests	R

Draw the arrow diagram of activities of the above project.

8. An architect has been awarded a contract to prepare plans for an urban renewal project for Asia Ltd. The job consists of the following activities and their estimated times:

Activity	Description	Immediate Predecessors	Time (days)
A	Prepare preliminary sketches	—	2
B	Outline specifications	—	1
C	Prepare drawings	A	3
D	Write specifications	A, B	2
E	Run off prints	C, D	1
F	Have specification	B, D	3
G	Assemble bid packages	E, F	1

(a) Draw an arrow diagram for this project.

(b) Indicate the critical path, and calculate the total float and free float for each activity.

9. A research and development department of NASA is developing a new power supply for a console television set. It has broken the job down into the following for:

Job	Description	Immediate Predecessors	Expected Time (days)
A	Determine output voltages	—	5
B	Determine whether to use solid state rectifiers	A	7
C	Choose rectifier	B	2
D	Choose filters	B	3
E	Choose transformer	C	1
F	Choose chassis	D	2
G	Choose rectifier mounting	C	1
H	Layout chassis	E, F	3
I	Build and test	G, H	10

(a) Draw a critical path scheduling arrow diagram, identifying jobs letters and associating times with each. Indicate the critical path.

(b) What is the minimum time for completion of the project?

10. The following maintenance job has to be performed periodically on the heat exchangers in Asian refinery:

Task	Description	Precedence	Duration (hours)
A	Dismantle pipe connections	—	14
B	Dismantle header, closure, and floating head front	A	22
C	Remove tube bundle	B	10
D	Clean bolts	B	16
E	Clean header and floating head front	B	12
F	Clean tube bundle	C	10
G	Clean shell	C	6
H	Replace tube bundle	F,G	8
I	Prepare shell pressure test	D, E, H	24
J	Prepare tube pressure test and make the final reassembly	I	16

(a) Draw an arrow diagram for this project.

(b) Identify the critical path. What is its length?

(c) Find the total float and free float for each task.

11. The sales manager of Export Products Limited was informed by the R&D department about the completion of the prototype of a particular product. He consulted the production manager on the time taken to produce the first batch of the product, which is needed for demonstration in his sales promotion programme. He also decided to invite a few industrial representatives to the demonstration of this new product and through them to launch it in the market. The various activities involved in this marketing project, their descriptions, estimated duration (in days) and immediate predecessors are given in the following table:

Activity	Description	Duration (days)	Immediate Predecessors
A	Collect data on specifications and capabilities	4	—
B	Prepare operation manual	4	A
C	Chart out promotion programme	4	B
D	Make copies of manual and promotion material	9	B
E	Produce first batch for demonstration	16	B
F	Prepare list of press representatives	2	C
G	Chief executive's conference with Managers	1	C
H	Press representatives reach Bombay	2	F,G
I	Promotional meetings	4	D, H
J	Product demonstration	2	E, I
K	Press representatives return home	2	J

(a) Draw the network diagram for the given project.

(b) Identify the critical path. What is the maximum time required to complete the project.

(c) Find the total float and free float (if any) for all the non-critical activities.

12. Listed in the table are the activities and sequencing requirements necessary for the completion of a research report of Kavi Ltd.

Activity	Description	Precedence	Duration (weeks)
A	Literature search	—	6
B	Formulation of hypothesis	—	5
C	Preliminary feasibility study	B	2
D	Formal proposal	C	2
E	Field analysis	A, D	2
F	Progress report	D	1
G	Formal research	A, D	6
H	Data collection	E	5
I	Data analysis	G, H	6
J	Conclusions	I	2
K	Rough draft	G	4
L	Final copy	J, K	3
M	Preparation of oral presentation	L	1

(a) Draw a network diagram for this project.

(b) Find the critical path. What is its length?

(c) Find- the total float and the free float for each non-critical activity.

13. Listed in the table are the activities and their simplified sequencing requirement for publishing a textbook of Ravi Ltd.

Activity	Description	Preceding Activity	Expected Completion Time (months)
A	Write book	—	12
B	Design book	A	1
C	Edit manuscript	A	6
D	Checking editing	C	2
E	Accept design	B	1
F	Copy edit	D, E	2
G	Prepare artwork	D, E	4
H	Accept and correct artwork	G	1/2
I.	Set galleys	F	4
J	Check and correct galleys	I	1

K	Full page proofs	H, J	2
L	.Check and correct pages	K	1
M	Prepare index	K	1
N	Set and correct index	M	1/2
O	Check camera-ready copy	L, N	1/2
P	Print and bind book	O	1

(a) Construct the PERT network and find the critical path.

(b) For each non-critical activity, find total float and free float.

14. A project of Kant Ltd. has the following activities and other characteristics:

Activity	Preceding Activity	Time Estimates (weeks)		
		Optimistic	Most Likely	Pessimistic
A	—	4	1	16
B	—	1	5	15
C	A	6	12	30
D	A	2	5	8
E	C	5	11	17
F	D	3	6	15
G	B	3	9	27
H	E, F	1	4	7
I	G	4	19	28

(a) Draw the PERT network diagram.

(b) Identify the critical path.

(c) Prepare the activity schedule for the project.

(d) Determine the mean project completion time.

(e) Find the probability that the project is completed in 36 weeks.

(f) If the project manager wishes to be 99 per cent sure that the project is completed on June 30, 1991, when should he start the project work?

15. A small project of Urvish Ltd., consists of seven activities, **the details of which are** given below:

Activity	Duration (days)			Immediate Predecessor
	Most Likely	Optimistic	Pessimistic	
A	3	1	1	—
B	6	2 .	14	A
C	3	3	3	A
D	10	4	22	B, C
E	7	3	15	B
F	5	2	14	D, E
G	4	4	4	D

(a) Draw the network, number the nodes, find the critical path, the expected project completion time and the next most critical path.

(b) What project duration will have 95 per cent confidence of completion?

16. The owner of a chain of Havmor fast food restaurants is considering a new computer system for accounting and inventory control. A computer company sent the following information about the computer system installation:

Activity	Activity Description	Immediate Predecessor	Times (days)		
			Optimistic	Most Likely	Pessimistic
A	Select the computer model	—	4	6	8
B	Design input/output system	A	5	1	15
C	Design monitoring systems	A	4	8	12
D	Assemble computer hardware	B	15	20	25
E	Develop the main programmes	B	10	18	26
F	Develop input/output routines	C	8	9	16
G	Create database	E	4	8	12
H	Install the system	D, F	1	2	3
I	Test and implement	G, H	6	7	8

(a) Construct PERT network diagram for this problem.

(b) Determine the critical path and compute the expected completion time.

(c) Determine the probability of completing the project in 55 days.

17 .Asia Ltd. is manufacturing plant and equipment for chemical plant, is in the Activity of quoting a tender called by a public sector undertaking. Delivery data once promised is crucial and penalty clause is applicable. Project manager has listed down the activities in the project as under:

Activity	Immediate Predecessor	Activity Time (weeks)		
		Optimistic	Most Likely	Pessimistic
A	—	1	3	5
B	—	2	4	6
C	A	3	5	7
D	A	5	6	7
E	C	5	7	9
F	D	6	8	10
G	B	7	9	11
H	E, F, G	2	3	4

Using PERT: (a) find out the delivery week from the date of commencement of the project, and (b) total float and free float for each of the non-critical activities.

18. The required data for a small project of Bharat Ltd. consisting of different activities are given below:

Activity	Predecessor Activities	Normal Duration Expense	Crash Duration Expense
A		(days) (Rs) 6 300	(days) (Rs) 5 400
B	—	8 400	6 600
C	A	7 400	5 600
D	B	12 1,000	4 1,400
E	C	8 800	8 800
F	B	7 400	6 500
G	D, E	5 1,000	3 1,400
H	F	8 500	5 700

(a) Draw the network and find out the normal project length and minimum project length, (b) If the project is to be completed in 21 days with minimum crash expense which activities should be crashed to how many days?

19. The time and expense estimates and precedence relationship of the different activities constituting a project of Smita Ltd. are given below:

Activity '	Predecessor Activities	Time (in weeks) Normal Crash	Expense (in Rs) Normal Crash
A		3 2	8,000 19,000
B	—	8 6	600 1,000
C	B	6 4	10,000 12,000
D	B	5 2	4,000 10,000
E	A	13 10	3,000 9,000
. F	A	4 4	15,000 15,000
G	F	2 1	1,200 1,400
H.	C, E, G	6 4	3,500 4,500
1	F	2 1	7,000 8,000

(a) Draw a project network diagram and find the critical path.

(b) If a dead line of 17 weeks is imposed for completion of the project, what activities will be crashed, what would be the additional expense and what would be critical activities of the crashed network after crashing?